Recitations 8

[Definitions used today]

- Correspondences: nonempty valued, single valued, closed valued, compact valued, convex valued, closed graph, convex graph, upper hemi-continuity, lower hemi-continuity, continuity.
- Sequential characterization of uhc and lhc., Berge (1963) maximum theorem

Question 1

Let $\Gamma : \Theta \rightrightarrows X$ be a correspondence.

1. Show that if a correspondence Γ has a closed graph then it is closed valued.
2. If Γ is compact valued and u.h.c then Γ has a closed graph.
3. If X is compact and Γ has a closed graph then Γ is u.h.c.

Question 2

Let consumer budget set at a price $p \in \Delta^{\ell}(p >> 0)$ and endowment e_i be

$$B(p, e_i) = \{x \in X_i : p \cdot x \leqslant p \cdot e_i\}$$

a Show that $B(p, e_i)$ is homogenous of degree zero in prices, non-empty valued and compact valued.

b Show that $B(p, e_i)$ is continuous.

Question 3

Let consumer i demand correspondence at a price p and endowment e_i be

$$x_i(p, e_i) = \left\{x \in B(p, e_i) : x_i \succeq_i y \quad \forall_{y \in B(p, e_i)}\right\}$$

a Show that if $B(p, e_i)$ is compact and $\succeq_i$ is complete and transitive preorder with upper contour sets $U_i(x) = \{y \in X_i : y \succeq_i x\}$ that are closed for all $x \in X$ then the demand is non-empty.

b Give an example illustrating that compactness is indeed a necessary condition.

Question 4

The consumer problem is often laid out without explicit endowments of the goods, instead the parameters are prices $p \in \mathbb{R}^l_{++}$ and a nominal income level $e \in \mathbb{R}_+$. The set of parameters is $\Theta = \mathbb{R}^l_{++} \times \mathbb{R}$. The **indirect utility function** and the **Marshalian demand correspondence** are:

$$v(p, e) = \max_{x \in B(p, e)} u(x) \quad x(p, e) = \{x \in B(p, e) \mid u(x) = v(p, e)\}$$

I take as given that B is a nonempty, convex valued and continuous correspondence, and that u is a continuous function. Show for v and x the following properties on Θ.

a v is a continuous function on Θ and x is a nonempty, compact valued, u.h.c. correspondence.

b v is nondecreasing in r for fixed p and non-increasing in p for fixed xe.

c v is jointly quasi-convex on (p, e).

d If u is (quasi) concave then v is (quasi) concave in e for fixed p.

e If u is (quasi) concave then x is a convex valued correspondence.

f If u is strictly (quasi) concave then x is a continuous function.