Recitations 24

Question 1 VCG [Final 2014]

There are n individuals considering whether or not to place a sculpture (already sculptured so it costs nothing) in the park. Each individual i has utility function:

$$u_i(k,t) = v_i k + t_i$$

for some number $v_i \in \mathbb{R}$ that reflects the taste of an individual for the sculpture, where $k \in \{0,1\}$ is an indicator of statue being placed in the park, the quantity $t_i \in \mathbb{R}$ is money transfer and $t \in \mathbb{R}^n$ is the money allocation.

1. Describe a Vickrey-Clarke-Groves (VCG) mechanism for this economy formally and intuitively.
2. Suppose $v_i \sim N(0,1)$. Show that for every individual i the probability that i will be pivotal (i.e. will end up paying some money) converge to 0 as $n \to \infty$.

Question 2 [88 IV.3 Spring 2009 majors]

Consider a quasilinear environment where two agents are to contribute to a public project. Let $K = \{0,1\}$ be the possible levels of the project, with 1 meaning that the project is "done", and 0 "not done". Agent i's private valuation of the project is denoted by θ_i, which is independently drawn from a uniform distribution on $[0,1]$. The project costs c to finish, where $0 < c < 2$. Let $k : \Theta \to K$, where $\Theta = \Theta_1 \times \Theta_2 = [0,1] \times [0,1]$, denote the following allocation function:

$$k(\theta_1, \theta_2) = \begin{cases} 1 & \text{if } \theta_1 + \theta_2 \geq c \\ 0 & \text{otherwise} \end{cases}$$

A transfer rule $t : \Theta \to \mathbb{R}^2$ specifies, for each agent i, the amount of monetary transfer received by agent i at each $\theta = (\theta_1, \theta_2) \in \Theta$. Writing $t(\theta)$ as $(t_1(\theta), t_2(\theta))$, we say that the transfer rule t balances the budget if, for any θ

$$t_1(\theta) + t_2(\theta) = \begin{cases} -c & \text{if } k(\theta) = 1 \\ 0 & \text{otherwise} \end{cases}$$

Construct a budget-balancing transfer rule t that Bayesian-implements the allocation rule k; i.e., construct a budget-balancing transfer rule t such that the social choice function (k,t) is Bayesian incentive compatible.

Question 3 [Final 2015]

A seller owns two objects, A and B, and decides to run a marginal product mechanism to sell them. In other words, she solicits from each bidder i his utility function $u_i = (u_i(A), u_i(B), u_i(AB)) \in \mathbb{R}^3$ (where $u_i(A)$ is i 's utility for good A, $u_i(B)$ is i 's utility for good B, $u_i(AB)$ is i 's utility for the bundle consisting of both goods A and B. Normalize $u_i(\emptyset) = 0$), computes the efficient allocation of objects given the bidder's reported utility functions and charges them an amount of money such that each bidder's net payoff always equals his marginal product if preferences are reported truthfully. Everyone has quasi-linear preferences with respect to money.

1. Suppose that there are two bidders with valuation: Completely derive the outcome of the auction in this case.

	A	B	AB
Bidder 1	0	0	12
Bidder 2	10	10	10

2. Suppose now instead there are three bidders with valuations: Completely derive the outcome of the

	A	B	AB
Bidder 1	0	0	12
Bidder 2	10	10	10
Bidder 3	10	10	10

auction in this case.

3. Suppose that the seller does not know whether there are two bidders with preferences like in i) or three bidders with preferences like in ii). Suppose that there are in fact only two bidders, as in i), but bidder 2 is able to pretend to be two separate bidders. What would be bidder's 2 decision and payoff in this case, assuming he chooses optimally whether or not to pretend to be two separate bidders?

4. Repeat three point above assuming that bidder's 1 preferences are (6,6,12) instead if $(0, 0, 12)$. Conjecture a reason what happens in iii) under the two assumptions on bidder's 1 utility function.