Previous midterms

Question 1 [Midterm 2017]

A normal form game has I players, action sets A^i and utility functions u^i, for $i \in I$. The vector $u^i : i = 1, \ldots, n$ is a utility profile. Fix the set of players and the action sets. The Nash equilibrium correspondence (NEC) associates to every utility profile the set of Nash equilibria of the normal form game with that profile.

a) Prove that one can restrict the domain of the correspondence to a compact sel of ntility profiles.

b) Prove that the NEC is closed valued.

c) Prove that the NEC is upper-hemi-continuous, or find an example to show it is not

Question 2 [Midterm 2017]

Define a Perfect Equilibrium of a normal form game as the limit of sequences of Nash equilibria of a perturbed game.

a) Prove that a perfect equilibrium is a Nash equilibrium.

b) A strategy is fully mixed if it gives positive probability to all the actions in the action set of the player, and a strategy profile is fully mixed if the strategies of all players are fully mixed. Prove that a fully mixed strategy profile is perfect.

c) Give as simple sufficient condition for a Nash equilibrium to be perfect.

Question 3 [Midterm 2017]

As in question 1, fix the set of players and the action sets.

a) Identify the set of all transformations of the utility function of a player that leave the Best Response correspondence unchanged. Please note you will state and prove statement like "The set of transformations T (put the definition here) of the utility functions of player 1 is such that for all normal formal games with that player set and action sets the Best Response for the utility function is u^1 and for its transformation $T\left(u^1\right)$ are the same."

b) Prove your answer in detail.

Question 4 [Midterm 2017]

a) Consider the utility function of player 1 :

	l	c	r
T	7	5	1
M	1	4	3
B	4	1	7

What are the correlated strategies for which player 1 will want to follow the recommended action for each of his actions?

b) What are the set of correlated equilibria of a zero-sum game?

Question 1 [Midterm 2018]

You are told that the following game: has a unique equilibrium in mixed strategies. What are

	L	R
T	(a, b)	(c, d)
B	(e, f)	(g, h)

the conditions on the utilities $\{a, b, \ldots\}$ that are necessary and sufficient for this statement to be true?

Question 2 [Midterm 2018]

a) State and prove precisely the relation between the two best response correspondences of player i, $\text{BR}^i_{A^i}$ and $\text{BR}^i \equiv \text{BR}^i_{\Delta(A^i)}$

b) Prove that the two correspondences are closed valued.

c) Consider the case when the action sets of all players A^i are countable. Can you identify conditions on the utility functions of the players so that the best response correspondence $\text{BR}^i_{\Delta(A^i)}$ is non-empty valued, convex and upper-hemicontinuous?

Question 3 [Midterm 2018]

a) Prove that the set of Nash equilibria of a normal form game with finite actions is closed.

b) Show that the set of Nash equilibria of a normal form game with finitely many players may be empty if one of the players has a countably infinite set of actions.

Question 4 [Midterm 2018]

a) Define a Perfect Equilibrium of a normal form game as the limit of sequences of Nash equilibria of a perturbed game.

b) Prove that a perfect equilibrium is a Nash equilibrium.

c) A strategy is fully mixed if it gives positive probability to all the actions in the action set of the player, and a strategy profile is fully mixed if the strategies of all players are fully mixed. Prove that a fully mixed strategy profile is perfect.

d) Give as simple sufficient condition for a Nash equilibrium to be perfect.

Question 1 [Midterm 2019]

The outcome of an Iterated Eliminations of Strictly Dominated Strategies (IESDS) in a normal form game is unique. In contrast, the outcome of an Iterated Eliminations of Weakly Dominated Strategies (IEWDS) is not necessarily unique, and depends on the consequence of eliminations. A crucial lemma in the proof of the statement on IESDS states, informally, that if an action can be eliminated at an earlier stage in an IESDS, then it can also be eliminated at a later stage.

a) Give a precise statement of the lemma.

b) Clearly, the statement of the lemma is false when we consider IEWDS. State clearly where in the proof of the lemma use is made of the fact that dominance is strict in the elimination procedure, and why the step fails in the IEWDS case.

Question 2 [Midterm 2019]

The following result is used in the proof of the existence of Nash equilibria. Let for $i = 1, \ldots, n$ $F^i : S \rightrightarrows S^i$ be a correspondence from S to S^i, where $S^i \equiv \Delta\left(A^i\right)$ as usual, and let F be the product of the correspondences.

a) Prove that F is uhc if each F^i is;

b) Prove that F is closed valued if each F^i is;

c) Prove that F is convex valued if each F^i is;

d) Is the result true when the action set is countable?

Question 3 [Midterm 2019]

Provide an example of a finite normal form game where the set of Nash equilibria

a) (1) is not finite and (2) for no player the set of mixed strategies used in some Nash equilibrium is the entire set of mixed strategies of the player. The condition (2) is important so we write it explicitly as:

b) if and only if $\nexists i \in I$ such that $\cup_{s=(s^i,\ldots,s^i,\ldots,s^n)\in NE} s^i = \delta\left(A^i\right)$ In particular you cannot use the game where all actions profiles give the same utility.

Question 4 [Midterm 2019]

In the following, fix the set of players I and the action set A^i of each player $i \in I$ for all normal form games considered.

a) Identify the set of all transformations of the utility function of a player that leave the Best Response correspondence in mixed strategies (this is the correspondence that we have denote $BR^i_{S^i}$ for each player unchanged.

b) Prove your answer in detail.

Question 1 [Midterm 2020]

Consider a normal form game with finitely many players and actions. Denote $\mathcal{U}$the set of all vNM utility representations (taking A to be the set of consequences) for player i and $\mathcal{U} \equiv \times_{i=1}^{n} \mathcal{U}^i$, the Cartesian product over the set of players

a) Prove that the correspondence from $\mathcal{U}$ to the set of Nash equilibria is well defined

b) Prove the correspondence is upper-hemi-continuous.

Question 2 [Midterm 2020]

Find all the solutions obtained by iterated elimination of weakly dominated strategies in the game:

	L	C	R
T	$1,2$	$2,3$	$0,3$
M	$2,2$	$2,1$	$3,2$
B	$2,1$	$0,0$	$1,0$

Question 3 [Midterm 2020]

The Beauty Contest is the following normal form game:

1. A set I of n players; the set $A^i \equiv \{0, 1, \ldots, 100\}$ (non-negative integers) for every $i \in I$;
2. The payoff is as follows.
   (a) For any $a \in A$, define $M(a)$ as the average value:
   $$M(a) \equiv \sum_{i=1}^{n} \frac{a^i}{n}$$
   and let $\theta \in (0,1)$ be fixed (for example, $\theta = 2/3$);
   (b) The player whose a^i is the unique closest to $M(a)\theta$ (note: the average times the θ factor). wins a desirable prize; the others get nothing.
   (c) If a tie at the closest position occurs, no one gets anything.
   (d) Does the value of θ matter, and if so how?

In this game:

1. Find the set of Nash equilibria.
2. Apply iterated elimination of strictly dominated strategies. What do you get?