# Recitations 16

**[Definitions used today]**

- Best correspondence, Nash Equilibrium, Minimax Theorem

## Question 1

| 1/2 | L | R |
|---|---|---|
| T | 3,1 | 0,0 |
| B | 0,0 | 1,3 |

- Define: pure actions, mixed actions, best correspondences
- Find all Nash Equilibria

## Question 2 [153 III.1 Spring 2013 majors]

A two players finite action normal form game is zero sum if the sum of the utilities of the two players is equal to 0 for any action profile, so $u^1 = -u^2$. **The Minimax Theorem** states that in this case

$$\min_{\alpha^2 \in \Delta(A^2)} \max_{\alpha^1 \in \Delta(A^1)} u\left(\alpha^1, \alpha^2\right) = \max_{\alpha^1 \in \Delta(A^1)} \min_{\alpha^2 \in \Delta(A^2)} u\left(\alpha^1, \alpha^2\right) \equiv v$$

Prove the minimax theorem. You can use Nash equilibrium existence theorem.

## Question 3

For a zero sum game of two players define the value of the game as $V : \mathbb{R}^{nm} \to \mathbb{R}$ (where $n = \#A^1$ and $m = \#A^2$) :

$$V(u) = \max_{s^1 \in \Delta(A^1)} \min_{s^2 \in \Delta(A^2)} U\left(s^1, s^2 \mid u\right)$$

where for a given strategy profile $s^1 = (p_1, \ldots, p_n)$, $s^2 = (q_1, \ldots, q_n)$ and payoffs $u \in \mathbb{R}^{nm}$ we define

$$U\left(s^1, s^2 \mid u\right) = \sum_{i=1}^{n} \sum_{j=1}^{m} p_i q_j u_{ij}$$

Show that **The value of a game** is

a) continuous

b) non-decreasing

c) homogenous of degree one in payoffs.

## Question 4

Under standard assumptions, prove the following properties of best response in mixed $\mathrm{BR}_i(s)$ :

a) non-empty valued,

b) compact valued,

c) upper hemi continuous.

d) convex-valued

**Question 5** Show that $\mathrm{BR}_i(s) = \mathrm{co}\left(\left\{\delta_{b^i} : b^i \in \mathrm{BR}^i_{A^i}(s)\right\}\right)$