Recitation 1

[Definitions used today]

- (strictly) convex, concave, quasi convex, quasi concave functions
- production set Y , input requirement set V, transformation function T, production function f
- DRS, IRS, CRS of production function
- NIRS, NDRS, CRS of production set
- Meet and Joint, Lattice, Supermodularity of a function, Increasing Differences function

Question 1 [Production function/set]

(a) Show that if $f(x)$ is concave $\Rightarrow$ production set Y is convex.

(b) Prove that for a convex production set $Y \Rightarrow$ input requirement set V is convex. Prove that converse is not true.

(c) Show that $f(x)$ is quasi concave function $\iff$ input requirement set V is convex.

(d) Show that if $f(x)$ is strictly concave and $f(0) = 0 \Rightarrow f$ exhibits DRS

Question 2 [Properties of Y, f]

Let $f(x)$ be a production function and Y a production set associated with f. Show the following propositions hold

(a) if f exhibits DRS then Y exhibits NIRS

(b) if f exhibits IRS then Y exhibits NDRS

(c) if f exhibits CRS then Y exhibits CRS

Question 3 [Supermodularity] 89 [I.1 Fall 2009 majors]

Show that following functions are **supermodular**

(a) the Cobb-Douglas production function $f(x) = x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}$, where $\forall_i\ \alpha_i > 0$, and $\sum_i \alpha_i < 1$

(b) the Leontief function $f(x) = \min_i\{\alpha_i x_i\} \quad \forall_i\ \alpha_i > 0$

Question 4 [Properties of Y]

Prove following properties

(a) Assume that for Y closed and convex , $Y \subset \mathbb{R}^L$ s.t. $0 \in Y$. Free disposal property $Y - \mathbb{R}^L_+ \subset T$ $\iff$ $\mathbb{R}^L_- \subset Y$

(b) If $y \in Y$ is profit maximizing for some $p \gg 0$, then y is efficient

(c) If Y is a convex set, then supply correspondence $s^*(p)$ is a convex set.

Question 5 165 [I.1 Fall 2013 minors]

Consider a production function $f : \mathbb{R}^n_+ \to \mathbb{R}_+$ with n inputs and one output. Assume that $f(0) = 0$.

(a) State a definition of f having (strictly) IRS.

(b) Prove that if f exhibits IRS, then, for any strictly positive input prices w_i (where $i = 1, \ldots, n$) and strictly positive output price p, either the firm's output at the profit-maximizing production plan is zero or otherwise the profit-maximizing production plan is not well defined (i.e. it does not exist).

(c) Consider the following example of production function with two inputs:

$$f(x_1, x_2) = [\min\{x_1, x_2\}]^2$$

Does this f exhibit increasing returns to scale?

(d) Does the cost-minimization problem for production function f of (c) have a solution for arbitrary prices $w_1 > 0, w_2 > 0$ and output level $y > 0$? Justify your answer